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AUTHOR	McNeil, Keith; Newman, Isadore	
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ABSTRACT

Analyses were conducted to determine whether the two statistical tests of significance are identical. Such demonstrations regarding parallel correlational and analysis of variance procedures have brought further understanding to each of the domains in the past. When a researcher is investigating the relationship between two variables and thinks that the relationship may be different for two subpopulations under consideration, that question can be tested. Some texts present this test as the test of significance for the difference between two correlations. The test uses the Fischer "z" transformation on the two sample correlations. The interaction between a dichotomous variable and a continuous variable is often of interest and can be tested within the general linear model approach. Two examples in which the z and F results are the same, and two in which they are different are presented. For most practical purposes, the two conceptually similar tests of significance actually test slightly different questions. The research hypothesis should be stated clearly to assure use of the right test of significance. (Contains 2 tables, 4 figures, and 16 references.) (SLD)

When the Difference Between Two Correlations Can be Tested by the Interaction between Continuous Variable and a Dichotomous Variable

Keith McNeil
New Mexico State University

Isadore Newman
Akron University

Purpose

The purpose of this investigation was to determine when the two statistical tests of significance are identical. Such demonstrations regarding parallel correlational and analysis of variance procedures have, in the past, brought further understanding to each of the domains (McClendon, 1994; McNeil & Beggs, 1969; McNeil, Newman, & Kelly, 1996; Ward & Jennings, 1973; Williams, 1974).

Inspiration for the Investigation

Several years ago the second author intimated, during a presentation at the American Educational Research Association, that the two tests of significance were the same (Newman, personal communication). While this made sense to the first author, investigation of numerous statistical texts has resulted in not discovering any such demonstration.

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Difference Between Two Correlations

When a researcher is investigating the relationship between two variables, and suspects that the relationship may be different for two subpopulations of the population under consideration, that question can be tested. Some statistical texts present this test as the test of significance for the difference between two correlations, although not all texts discuss this test (e.g., Gravetter & Wallnaw, 1992; Pagano, 1994; McClave & Benson, 1998). The test uses the Fischer *z* transformation on the two sample correlations as the sampling distribution for population correlations (other than a population correlation of 0) will not approach normality. The test only depends on the sample correlations and the size of the samples. The test is a *z* test as follows:

$$z = \frac{(z_{r1} - z_{r2}) - \text{(the hypothesized difference between the two correlations)}}{\text{square root of } 1 / [(n_1 - 3) + 1/(n_2 - 3)]}$$

Three degrees of freedom are lost for each sample because the Fischer z transformation is used. The hypothesized difference between the two correlations will often be zero, as in all of the examples in this paper. The hypothesized difference between the two correlations would not have to be zero, and that would be a natural extension of this investigation (see McNeil, 1991 for non-zero restrictions on other statistical tests). This test is presented in such tests as Hinkle, Wiersma, and Jurs (1994, p. 263). Figure 1 is a plot of the data used for case 1. A “1” indicates data from group 1 and a “2” indicates data from group 2. Analysis of the data in Figure 1 is the limiting case when the slopes are the same and the standard deviations with the groups are the same. Other analyses consider same slopes with different standard deviations, and different correlations with same slopes.

Interaction Between a Dichotomous Variable and a Continuous Variable

While most statistics texts provide the impression that interaction is only of interest in guiding the researcher to either simple effects or main effects (such as in a

two-way design), interaction may be of interest in its own right. For example, the area of aptitude by treatment interaction, and matching the instructional style with the learning style are two such areas of investigation wherein interaction is of primary importance. Furthermore, the discovery of interaction may help clear up conflicting results in a given area (Bender, Kelly, & Pierson, 1968; McNeil & Newman, 1995; McNeil, Newman, & Kelly, 1996).

Interaction can probably best be understood by reference to Figures 1 and 2. Figure 1 depicts the state of affairs wherein there is no interaction. Figure 2 depicts the interaction between the dichotomous variable (say, teaching style) and the continuous variable of X (say, aptitude). The suspected interaction can be tested within the General Linear Model approach (GLM) by developing a Full Model which allows two lines of best fit (one for group 1 and one for group 2):

Full Model (Two lines): $Y = a_1 U_1 + b_1 X_1 + a_2 U_2 + b_2 X_2 + E_1$

where: U_1 = 1 for participants in group 1, 0 otherwise,

X_1 = the continuous value of X for participants in group 1, 0 otherwise,

U_2 = 1 for participants in group 2, 0 otherwise,

X_2 = the continuous value of X for participants in group 2, 0 otherwise,

and a_1, b_1, a_2, and b_2 are least squares weighting coefficients calculated so as to minimize the sum of the squared errors in each of the error vectors, E_1 and E_2. (a_1 and a_2 are the two intercepts and b_1 and b_2 are the two slopes.)

In order to test for interaction, one must see how the model of two parallel lines fits the data. Parallel lines means that the slopes of the two lines are the same. Therefore one must restrict the two slopes in the above "two line model" to be equal to a common slope, say "b." This results in the following Restricted Model:

Restricted Model (Two parallel lines): $Y = a_1 U_1 + b X_1 + a_2 U_2 + b X_2 + E_2$

but $[b X_1 + b X_2] = b X$, and therefore the above model is equivalent to:

Restricted Model (Two parallel lines): $Y = a_1 U_1 + b X + a_2 U_2 + E_2$
where b is the common slope for each of the two groups.

This test of interaction is presented in various GLM texts (Kerlinger & Pedhazur, 1973; McClendon, 1994; McNeil, Newman, & Kelly, 1996; Ward & Jennings, 1973). Since there are 4 predictor variables in the Full Model and 3 in the Restricted Model, there is 1 (4 - 3) degree of freedom in the numerator of the *F* test. The denominator degrees of freedom are the total number of participants minus the 4 pieces of information in the Full Model, or $n_1 + n_2 - 4$. Since $z^2 = F$ when there is one degree of freedom in the numerator, and an infinite degrees of freedom in the numerator, the two tests could produce the same answer under the condition of an infinite degrees of freedom in the denominator--when there is a very large number of participants in the two samples. The astute reader will realize that the degrees of freedom in the numerator for the two tests are indeed 1, but the degrees of freedom denominator are different, thus the two tests will usually not be exactly the same.

Case 1

In case 1, group 2 data was an exact replication of group 1 data (see Table 1). The means, standard deviations, and correlations within the two groups were exactly the same. The slopes were also exactly the same. Since there was no difference between the correlations, the *z* test was also equal to 0.00. Hence for case 1, the two tests of significance were identical.

Case 2

By adding a constant of 7 to all the X scores in group 1, the data for group 1 is moved 7 units to the right, as in Figure 1. The correlations are not affected (still .57) by this linear transformation, and the slopes are also not affected (still 1.00). The two tests of significance will each yield a value of 0.00, as indicated in Table 1 for case 2.

Case 3

Case 3 considers data that has an opposite correlation within the two groups,

as in Figure 2. As Table 1 indicates, the two tests of significance were quite discrepant for case 3.

Case 4

Case 4 considered the possibility of different standard deviations in the two groups. The data from case 2 was modified by making the deviations from the line of best fit within group 1 only one unit, rather than 2 units as in group 2 (as indicated in Figure 3). Clearly, the slopes are the same within each group, resulting in an *F* of 0.00, but the correlations are different (.81 vs. .57), resulting in a *z* value of .87.

Conclusion

The two tests of significance are isomorphic only when within the two groups there is the same correlation, the same variance on X, and the same variance on Y. Except in the limiting cases, the two tests are testing different questions. If the question is about slopes, irrespective of variances, the interaction test is appropriate. If the variances within groups are of concern, then the correlation test is appropriate. One should note that a correlation coefficient is independent of means and variances. Thus the two groups, while allowed to have these differences, are in a sense forced to have the same means and variances when the correlation test of significance is used. The interaction test of significance analyses the data the way it is, not forcing the two groups to have the same means and variances. Thus, for most practical purposes the two tests of significance, while conceptually similar, actually test slightly different questions. Therefore, one test is not better than the other, they simply test similar, but different questions (except in the trivial situation of case 1). If one does not clearly state the research hypothesis, one might use the wrong test--resulting in a Type VI error as discussed by Newman and Newman (1994).

Epilogue I

In discussing the rough draft of the present paper, we decided to investigate one additional case, wherein the data was standardized within each of the two groups, discussed below as case 5.

Case 5

Case 5 considers a data set (case 4 data) that has been standardized within each group. The group mean was subtracted from each X and Y score, and then divided by the appropriate standard deviation (creating *z* scores within each group). The correlations within the groups remain the same because this is a linear transformation, but the slopes are now different. Instead of slopes of 1.00 within each of the two groups, the slopes are now equal to the correlations (.81 and .57 respectively). The test for the difference between the two correlations was still a z of .87. The *F* test for linear interaction was .51. We had expected the two results to be the same (that is $z*z = F$).

Going back to Fisher (1941), it is claimed that the Fisher *z* transformation is approximately normal, and is appropriate for moderately sized samples. Therefore we investigated larger sample sizes. Table 2 contains the results for *N* of 20, 40, 80, 160, and 320 per group. Note that as the sample size becomes larger, that $z*z$ is more and more closely equal to twice that of the *F* test, though we had expected the two to become similar.

Epilogue II

In discusssing these results amonst ourselves, we have concluded that the reason that we are getting different results is indeed because different questions are being asked. The interaction question is actually testing the partial correlations, whereas the Fisher z test is testing the correlations. Similar questions, but different answers.

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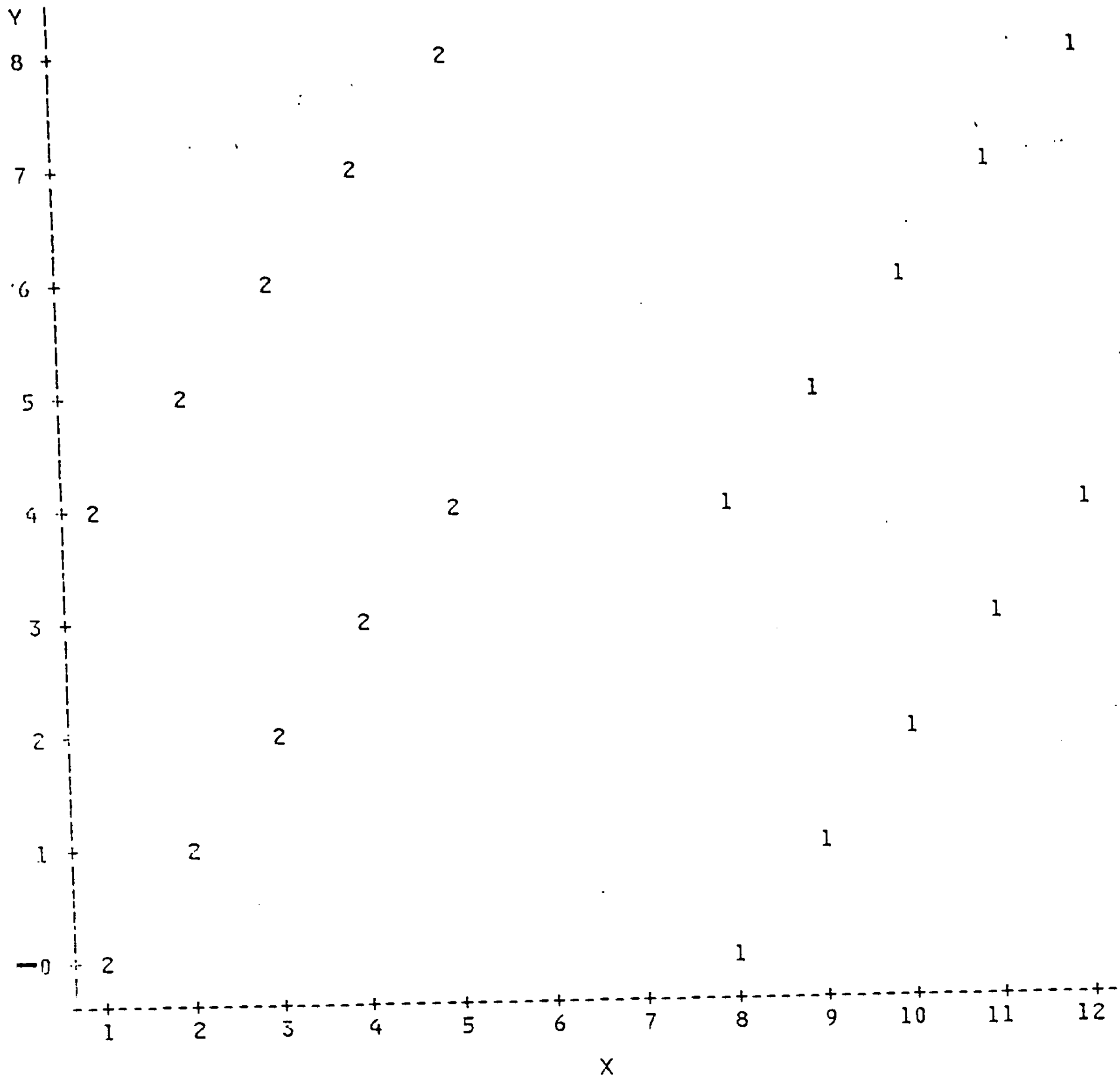
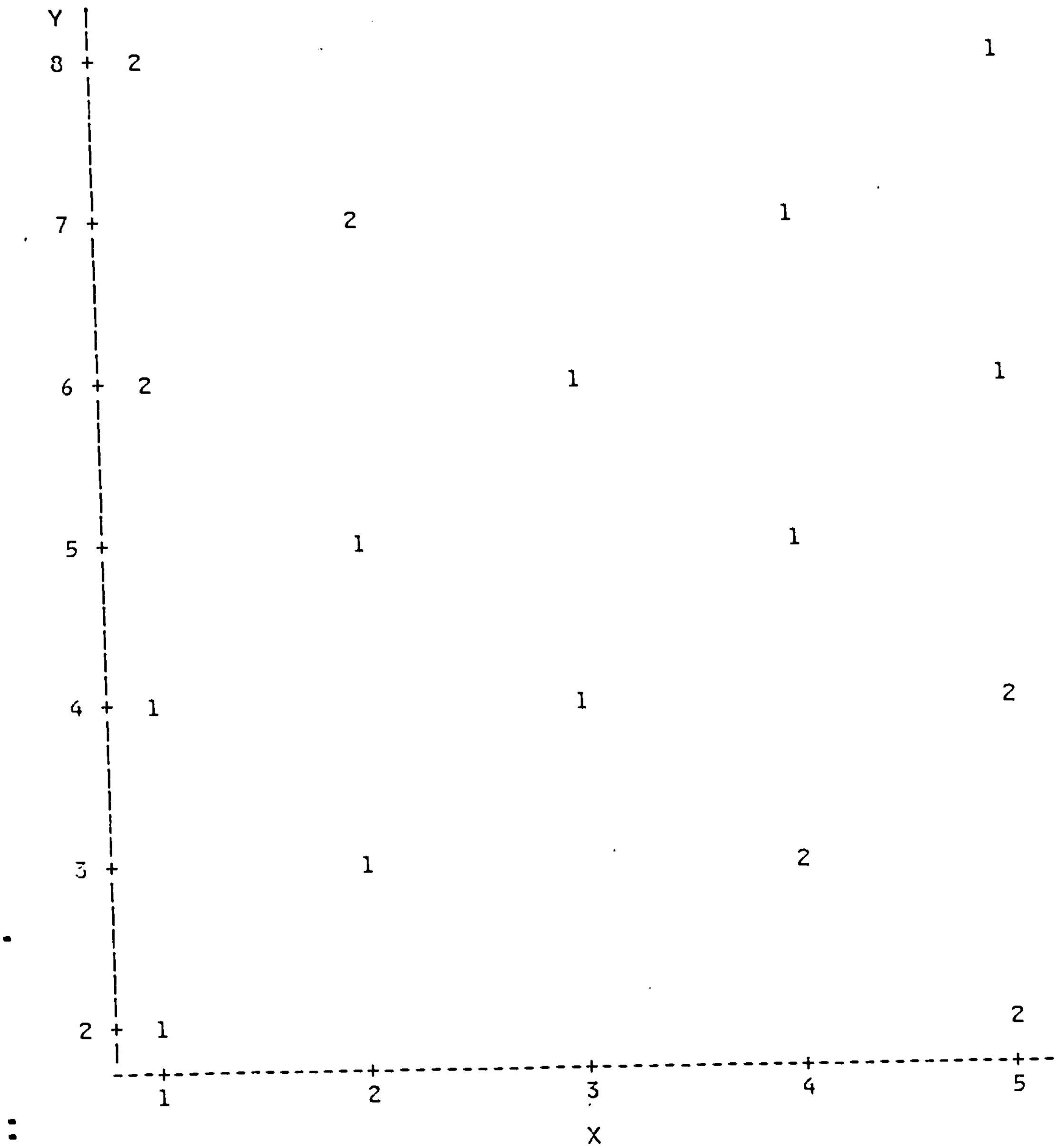


Figure 1. No interaction, with data in group 1 7 units higher on X than data in group 2.



NOTE: 4 obs hidden.

Figure 2. Interaction between group and X.

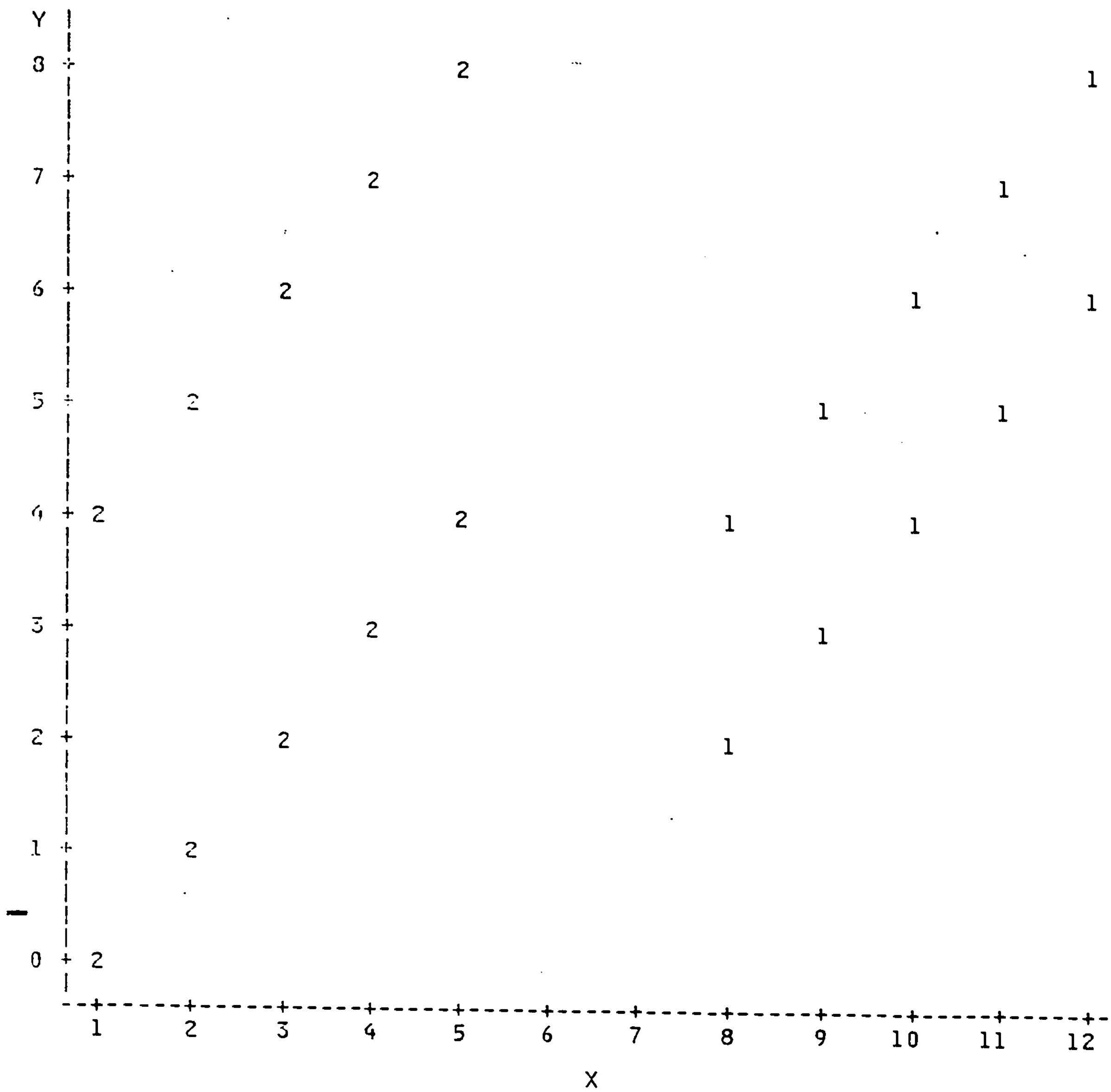


Figure 3. No interaction (parallel slopes), but less error variance in group 1, and hence higher correlation between X and Y in group 1 than in group 2.

```
OPTIONS LS=70;
TITLE DIFFERENCE BETWEEN TWO CORRELATIONS--SIGNIFICANT INTERACTION;
TITLE2;
TITLE3 ONE GROUP OFFSET 7 POINTS;
TITLE5 DIFFERENT CORRS, SAME SLOPE;
OPTIONS LINESIZE = 72;
DATA NEW;
INPUT X Y GROUP;
G1 = 0;   G2 = 0;  XSL1  = 0;  XSL2 = 0;
IF GROUP = 1 THEN G1 = 1 ;  IF GROUP = 1 THEN X = X+7;
IF GROUP = 2 THEN G2 = 1 ;
SL1 = X * G1;
SL2 = X * G2;
CARDS;
1 2 1
1 4 1
2 3 1
2 5 1
3 4 1
3 6 1
4 5 1
4 7 1
5 6 1
5 8 1
1 0 2
1 4 2
2 1 2
2 5 2
3 2 2
3 6 2
4 3 2
4 7 2
5 4 2
5 8 2
 PROC SORT; BY GROUP;
 PROC CORR; BY GROUP;
 PROC REG; MODEL Y =  SL1  SL2 G1 ;
 TEST  SL1 =  SL2;
 PROC PLOT; PLOT Y * X =GROUP;
```

Figure 4. SAS setup for data in Figure 3.

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Table 1

Results from the Various Analyses

	Group 1					Group 2					Statistics	
Case	M_x	M_y	S_x	S_y	r	M_x	M_y	S_x	S_y	r	z	F
1	3	4	1.49	1.49	.57	3	4	1.49	1.49	.57	0.00	0.00
2	10	4	1.49	2.58	.57	3	4	1.49	2.58	.57	0.00	0.00
3	3	5	1.49	1.82	.81	3	5	1.49	1.82	-.81	4.21	32.00
4	10	5	1.49	1.82	.81	3	4	1.49	2.58	.57	.87	0.00

Note. N in both groups is always 10.

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Table 2

Results from Different Sample Sizes

Sample Size per Group	z	$z^{*}z$	F
10	.87	.75	.51
20	1.73	2.97	1.03
40	2.02	4.07	2.17
80	2.91	8.46	4.46
160	4.34	18.84	9.04
320	6.09	37.13	18.22

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Printed Name: Keith McNeil	Organization: NMSU
Address: Box 3CEP NMSU Las Cruces NM 88003	Telephone Number: (505) 646 2124
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